
CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2019 Trial Examination

FORM VI

MATHEMATICS 2 UNIT

Monday 12th August 2019

General Instructions

- Reading time — 5 minutes
- Writing time — 3 hours
- Write using black pen.
- NESA-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I — 10 Marks

- Questions 1 – 10 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II — 90 Marks

- Questions 11–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your candidate number on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Eleven.

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Reference Sheet
- Candidature — 90 boys

Examiner

PKH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What is the value of $\log_e 5 + \log_e 50$, correct to two decimal places?

(A) 5·52

(B) 2·08

(C) 4·01

(D) 2·40

QUESTION TWO

What is the derivative of $y = e^{2x+1}$?

(A) $y' = (2x + 1)e^{2x+2}$

(B) $y' = e^{2x+1}$

(C) $y' = 2e^{2x+1}$

(D) $y' = \dfrac{1}{2}e^{2x+1}$

QUESTION THREE

What is the equation of the line parallel to the y-axis passing through the point $(-3, 5)$?

(A) $x = -3$

(B) $x = 5$

(C) $y = -3$

(D) $y = 5$

QUESTION FOUR

What is the natural domain of the function $y = \dfrac{1}{\sqrt{2x+1}}$?

(A) $x > -\dfrac{1}{2}$

(B) $x < -\dfrac{1}{2}$

(C) $x \le -\dfrac{1}{2}$

(D) $x \ge -\dfrac{1}{2}$

Examination continues next page ...

QUESTION FIVE

Which of the following is equivalent to $\dfrac{1}{2\sqrt{5}+3}$?

(A) $\dfrac{2\sqrt{5}+3}{-11}$

(B) $\dfrac{2\sqrt{5}-3}{17}$

(C) $\dfrac{2\sqrt{5}-3}{11}$

(D) $\dfrac{2\sqrt{5}+3}{11}$

QUESTION SIX

Which expression is equivalent to $3^x \times 2^x + 3x^{-2}$?

(A) $6^{2x} + \dfrac{1}{3x^2}$

(B) $6^x + \dfrac{1}{9x^2}$

(C) $6^{2x} + \dfrac{3}{x^2}$

(D) $6^x + \dfrac{3}{x^2}$

QUESTION SEVEN

What are the correct solutions to the equation $e^{10x} - 11e^{5x} + 10 = 0$?

(A) $x = 1$ and $x = 10$

(B) $x = 1$ and $x = \log_e 10$

(C) $x = 0$ and $x = \log_e 10$

(D) $x = 0$ and $x = \dfrac{1}{5}\log_e 10$

QUESTION EIGHT

How many solutions does the equation $6x\left(\dfrac{1}{x}+1\right)=0$ have?

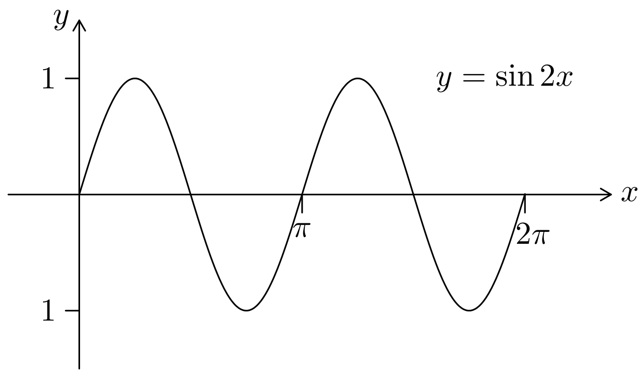
(A) 0
(B) 1
(C) 2
(D) 3

QUESTION NINE

Which is a correct expression for $-x^2-6x-5$?

(A) $4+(x+3)^2$
(B) $(x-3)^2-4$
(C) $4-(x+3)^2$
(D) $-4+(x+3)^2$

QUESTION TEN



How many solutions does the equation $5\sin 2x = x$ for $0 \le x \le 2\pi$ have?

(A) 2
(B) 3
(C) 4
(D) 5

End of Section I

Examination continues next page . . .

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Solve the inequation $2 - 3x \leq 11$. **1**

(b) For the arithmetic sequence $12, 6, 0, \ldots$ find the 41st term. **2**

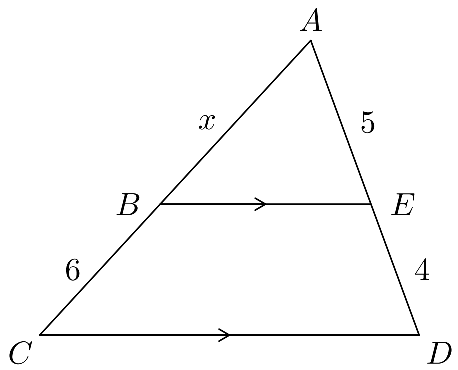
(c) Differentiate:

(i) $y = 3x - \dfrac{1}{x}$ **1**

(ii) $y = \tan(2x + 1)$ **1**

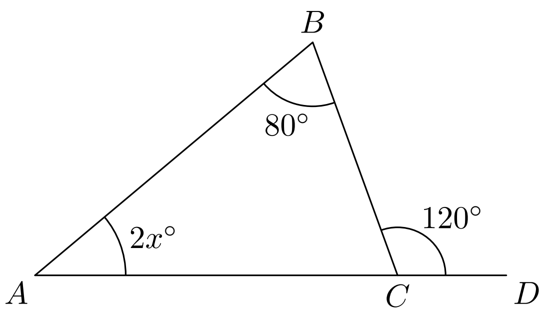
(iii) $y = \ln(x^2 + 1)$ **1**

(d) **1**



Find the value of x in the diagram above.

(e) **2**



Find the value of x giving reasons.

Examination continues overleaf ...

QUESTION ELEVEN (Continued)

(f) Find:

(i) $\int (x^2 - x + 2)\,dx$ **1**

(ii) $\int \sin(3x + 1)\,dx$ **1**

(g) Simplify $\dfrac{x^4 - x^2}{x - 1}$. **2**

(h) Differentiate $y = \dfrac{2x}{1 + x^2}$. **2**

Examination continues next page ...

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

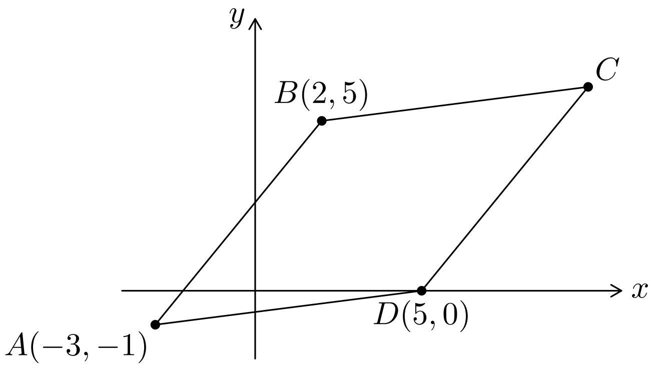
(a) Sketch the following functions, showing any intercepts or asymptotes. Sketch on separate axes.

(i) $y = -2x + 3$ **2**

(ii) $y = \dfrac{1}{x - 4}$ **2**

(iii) $y = 2^{-x} + 2$ **2**

(b)



In the diagram above, $ABCD$ is a parallelogram.

(i) Find the co-ordinates of C if C lies in the first quadrant. **2**

(ii) Find the equation of line DC. **2**

(iii) Find the distance from B to line DC. **2**

(iv) Find the area of parallelogram $ABCD$. **2**

(v) Is $ABCD$ a rhombus? Give a reason for your answer. **1**

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Solve the equation $5^{1-x} = 25^x$. **1**

(b) A geometric series has common ratio of $-\dfrac{2}{3}$ and a limiting sum of 12. **2**

Find the first term.

(c) Let R be the region bounded by the line $y = x$ and the parabola $y = 4x - x^2$

(i) Sketch and shade the region R and find the x co-ordinates of the points of intersection of the line and the parabola. **3**

(ii) Find the area of the region R. **2**

(d) Two men leave point P. One travels 12 km on a bearing of 40°T and stops at point A. The second man travels 15 km on a bearing of 280°T and stops at point B.

(i) Represent this situation on a diagram. **1**

(ii) Find the size of angle APB. **1**

(iii) Find the distance AB correct to the nearest 100 metres. **2**

(iv) Find the true bearing of A from B to the nearest degree. **3**

QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Solve the equation $2x^2 - 2x = 0$. **1**

(b) Solve $|2x - 3| = x$. **2**

(c) (i) Sketch $y = \tan x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. **1**

(ii) Find the equation of the normal to the curve $y = \tan x$ at the point where $x = \dfrac{\pi}{4}$. **2**

(d) The quadratic equation $2x^2 - 3px + 36 = 0$ has roots α and 2α, where $\alpha > 0$.

(i) Find the value of α. **1**

(ii) Find the value of p. **1**

(e) A parabola has equation $y^2 - 4y = 8x + 4$.

(i) Find the focal length and the vertex. **2**

(ii) Sketch the parabola showing the focus and directrix. **2**

(f) Solve the equation $\cos\theta - 2\cos\theta\sin\theta = 0$ for $0 \le \theta \le 2\pi$. **3**

Examination continues next page ...

QUESTION FIFTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Use Simpson's rule with 3 function values to approximate $\displaystyle\int_1^3 2^{x^2}\,dx$. **2**

(b) A particle is moving in a horizontal line and its displacement x metres from a fixed point O is given by $x = \dfrac{t^3}{3} - \dfrac{t^2}{2} - 6t + 11$ where t is measured in seconds. **3**
Find the average speed over the first 4 seconds.

(c) A solid of revolution is formed when the region bounded by $y = \ln x$, the x-axis and $x = e^2$ is rotated about the y-axis.

(i) Show the region to be rotated on a set of axes. **2**

(ii) Find the volume of the solid of revolution in exact form. **3**

(d) A university student plans to finish university in two years time. He wishes to travel overseas and thinks he will need \$15 000. He decides to invest \$$M$ at the start of each month in an investment account earning 3% per annum compounded monthly. Let A_n be amount in the account at the end of n months.

(i) Find A_2 in terms of M. **1**

(ii) Show that $A_n = 401M(1{\cdot}0025^n - 1)$. **2**

(iii) Find the amount he must invest each month to reach his target of \$15 000. Give your answer to the nearest dollar. **2**

Examination continues overleaf ...

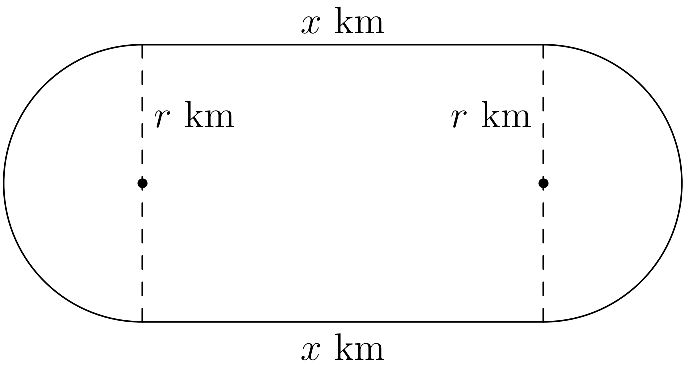
QUESTION SIXTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) The population P of a town was 12 500 at the beginning of the year 2000. Its population is growing at a rate proportional to its size, that is $\dfrac{dP}{dt} = kP$, where t is the time in years after 2000. At the beginning of 2015 its population was 16 000.

(i) Find the value of k. **2**

(ii) What will the population be at the beginning of 2021? **2**

(b)



A racetrack of perimeter one kilometre is formed from two equal semi-circles and a rectangle as shown in the diagram above. The straight sides have length x km and the circles have radius r km.

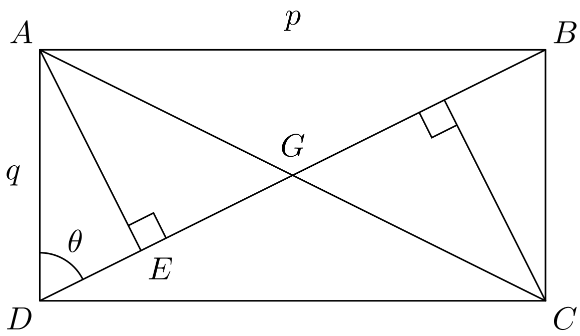
(i) Show that $x = \dfrac{1 - 2\pi r}{2}$. **1**

(ii) Show that the area of the rectangle is $A = r - 2\pi r^2$. **1**

(iii) Find the values of x and r for which the area of the rectangle is maximised. **3**

QUESTION SIXTEEN (Continued)

(c) The diagram below shows a rectangle $ABCD$ with $q < p$. Let $\theta = \angle ADE$.



(i) Show that $\cos\angle ADE = \dfrac{q}{\sqrt{p^2+q^2}}$. **1**

(ii) Show that $AE = \dfrac{pq}{\sqrt{p^2+q^2}}$. **2**

(iii) Show that the area of triangle AEG is $\dfrac{pq(p^2-q^2)}{4(p^2+q^2)}$. **3**

End of Section II

END OF EXAMINATION

SGS 2019 FORM VI MATHEMATICS 2 UNIT

Solution Section A (10 marks)

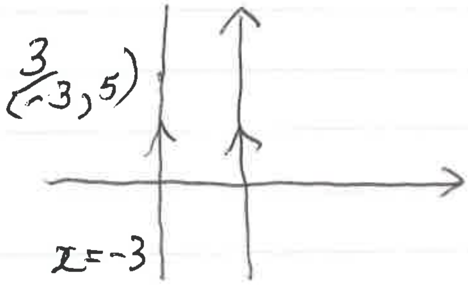
1 $\log_e 5 + \log_e 50$

$= 5.52$ (A)

2 $y = e^{2x+1}$

$y' = 2e^{2x+1}$ (C)

3 $(-3, 5)$



(A)

4 $2x + 1 > 0$

$x > -\frac{1}{2}$ (A)

5 $\frac{1}{2\sqrt{5}+3} \times \frac{2\sqrt{5}-3}{2\sqrt{5}-3}$

$= \frac{2\sqrt{5}-3}{20-9}$

$= \frac{2\sqrt{5}-3}{11}$ (C)

6 $3^x \times 2^x + 3x^{-2}$

$= 6^x + \frac{3}{x^2}$ (D) [redacted]

7 Let $e^{5x} = u$,

$u^2 - 11u + 10 = 0$

$(u - 10)(u - 1) = 0$

$e^{5x} = 10 \quad e^{5x} = 1$

$x = \frac{1}{5}\log_e 10 \quad x = 0$

(D)

8

$6x\left(\frac{1}{x} + 1\right) = 0$

$x = 0$ or $\frac{1}{x} = -1$

Not valid $\quad x = -1$

1 solution

(B)

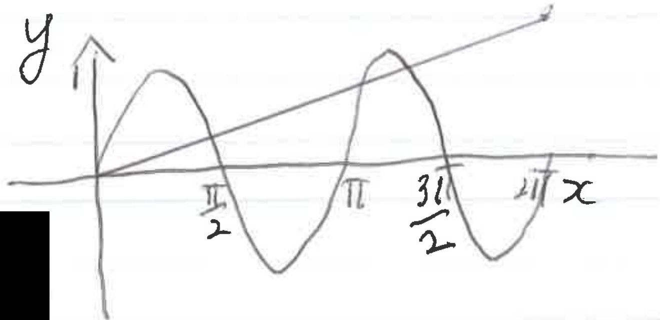
9 $-x^2 - 6x - 5$

$= -(x^2 + 6x + 5)$

$= -(x^2 + 6x + 9) + 4$

$= -(x + 3)^2 + 4$

$= 4 - (x + 3)^2$ (C)



$\sin 2x = \frac{1}{5}x$

$x = 2\pi \quad \frac{1}{5}2\pi = 1.25$

4 solutions (C)

ELEVEN (15 marks)

/15

(a) $2 - 3x \leq 11$

$-3x \leq 9$

$x \geq -3$ ✓

(b) $12, 6, 0, \ldots$

$a = 12, \; d = -6$ ✓

$T_n = a + (n-1)d$

$T_{41} = 12 + 40 \times -6$

$= -228$ ✓

(c) (i) $y = 3x - x^{-1}$

$y' = 3 + \frac{1}{x^2}$ ✓

(ii) $y = \tan(2x+1)$

$y' = \sec^2(2x+1) \times 2$

$= 2\sec^2(2x+1)$ ✓

(iii) $y = \ln(x^2+1)$ ✓

$y' = \frac{2x}{x^2+1}$

(d) $\frac{x}{5} = \frac{6}{4}$

$x = \frac{30}{4}$ ✓

$x = 7\frac{1}{2}$

(e) $2x + 80 = 120$

(Ext angle Th) ✓

$2x = 40$ ✓

$x = 20$

(f) (i) $\int x^2 - x + 2 \, dx$

$= \frac{x^3}{3} - \frac{x^2}{2} + 2x + C$

✓

(ii) $\int \sin(3x+1) \, dx$

$= -\frac{1}{3}\cos(3x+1) + C$

(g) $\frac{x^4 - x^2}{x - 1}$

$= \frac{x^2(x^2-1)}{x-1}$

$= \frac{x^2(x-1)(x+1)}{x-1}$ ✓

$= x^2(x+1)$ ✓

(h) $y = \frac{2x}{1+x^2}$

$y' = \frac{vu' - uv'}{v^2}$

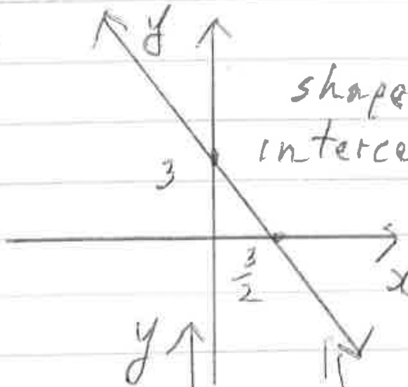
$= \frac{(1+x^2)2 - 2x \times 2x}{(1+x^2)^2}$

$= \frac{2(1-x^2)}{(1+x^2)^2}$

give marks here ✓ ✓

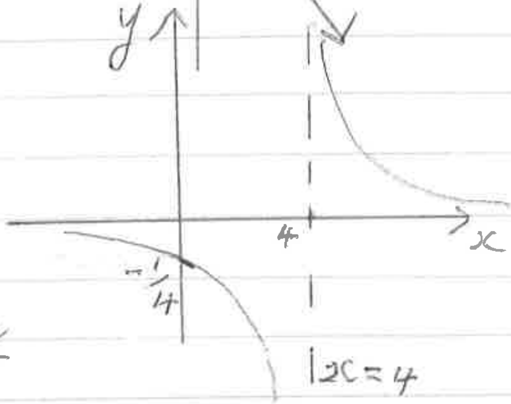
TWELVE

(a) (i)



shape ✓
intercepts ✓

(ii)

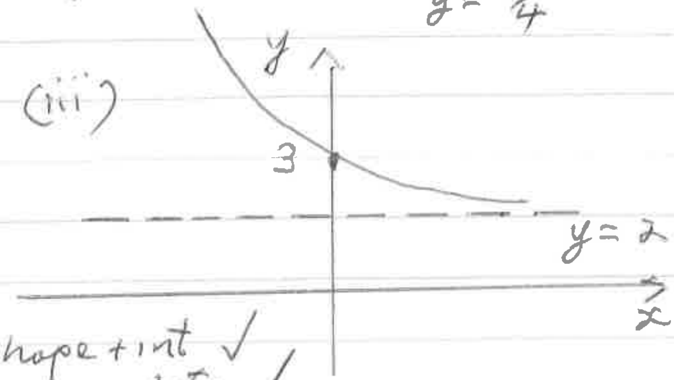


✓ shape + int

✓ asymptote

When $x = 0$
$y = -\frac{1}{4}$

(iii)



shape + int ✓
asymptote ✓

(b) (i) A → D 8 across 1 up ✓
B → C same

So $C = (10, 6)$ ✓

(ii)

$m(DC) = \frac{rise}{run} = \frac{6}{5}$ ✓

Eqn of DC is

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{6}{5}(x - 5)$ ✓

$y = \frac{6}{5}x - 6$

$6x - 5y - 30 = 0$

(iii) Dist ~~from~~ B to DC is $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

$(2, 5)$
(x_1, y_1)
$A = 6$
$B = -5$
$C = -30$

$= \frac{|6 \times 2 - 5 \times 5 - 30|}{\sqrt{6^2 + 5^2}}$ ✓

$= \frac{43}{\sqrt{61}}$ ✓

(iv) Area of ||gram

$= b \times h$

$= |DC| \times \frac{43}{\sqrt{61}}$ ✓

$= \sqrt{61} \times \frac{43}{\sqrt{61}}$

$= 43$ units^2 ✓

(v) $|AD| = \sqrt{8^2 + 1^2} \neq \sqrt{61}$
Adjacent sides are not equal ✓

15

THIRTEEN

④ $\frac{}{15}$

(a) $5^{1-x} = 25^{x}$

$1 - x = 2x$

$x = \frac{1}{3}$ ✓

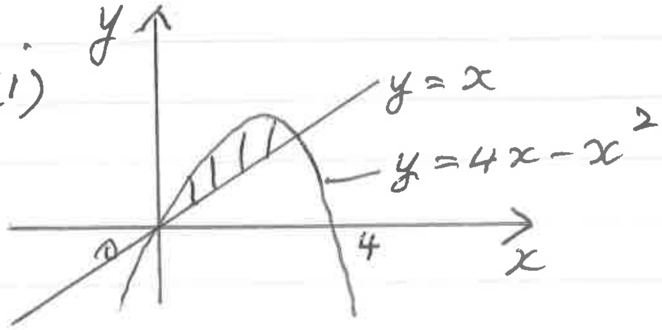
(b) $S_\infty = \frac{a}{1-r}$

$12 = \frac{a}{1+\frac{2}{3}}$ ✓

$12 = \frac{a}{\frac{5}{3}}$

$a = 20$ ✓

c (i)



✓ correct graphs
✓ correct shading

(ii) Points of intersection where

$x = 4x - x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$ ✓

$x = 0,\ 3$

(iii) $\text{Area} = \int_0^3 4x - x^2 - x\,dx$

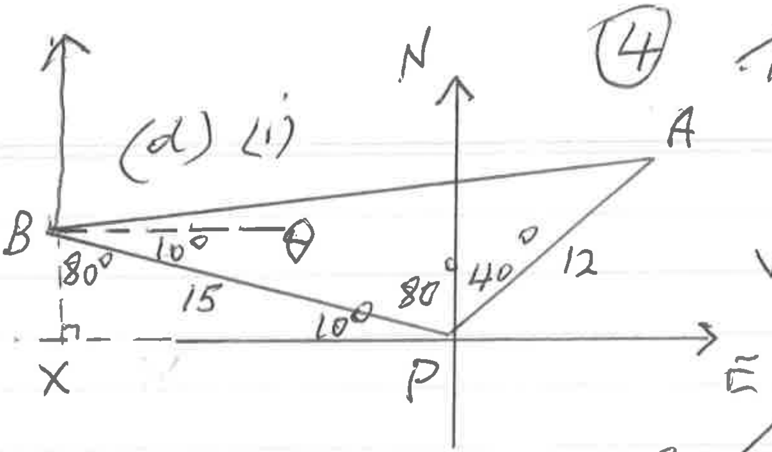
$= \int_0^3 3x - x^2\,dx$ ✓

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$

$= \frac{27}{2} - 9$ ✓

$= \frac{9}{2}$ units^2

(d) (i)



✓

(ii) $\angle APB = 120^\circ$ ✓

(iii) $AB^2 = 15^2 + 12^2 - 2 \times 15 \times 12 \cos 120^\circ$ ✓

$= 225 + 144 - 2 \times 180 \times -\frac{1}{2}$

$= 369 + 180$

$= 549$ ✓

$AB = \sqrt{549} \doteqdot 23.4$ km

(iv) See diagram

Let $\theta = \angle ABP$

$\cos\theta = \frac{AB^2 + BP^2 - AP^2}{2 \times AB \times BP}$ ✓

$\cos\theta = \frac{549 + 225 - 12^2}{2 \times 15 \times \sqrt{549}}$

$\theta \doteqdot 26.3^\circ$

Bearing of A from B ✓

$\doteqdot 180^\circ - 80^\circ - 26.3^\circ$

(Note $\angle XBP = 80^\circ$)

$= 74^\circ$T (to the nearest degree) ✓

[You can also use Sine rule]

FOURTEEN 15

(a) $2x^2 - 2x = 0$

$2x(x-2) = 0$

$x = 0$ or $x = 2$ ✓

(b) $|2x-3| = x$

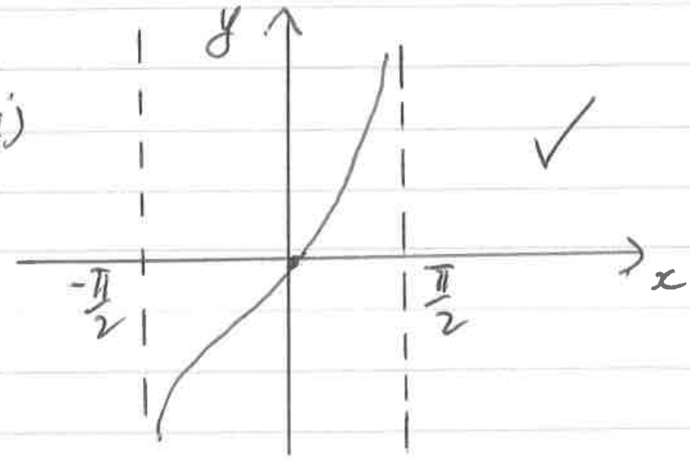
$2x-3 = x$ or $2x-3 = -x$

$x = 3$ or $x = 1$

✓ ✓

(c)

(i)



✓

(ii) $y = \tan x$

$y' = \sec^2 x$ ✓

When $x = \frac{\pi}{4}$, $M = \sec^2\frac{\pi}{4} = 2$

$M_1 = -\frac{1}{2}$, $y = 1$

Equ of the normal is ✓

$y - 1 = -\frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(d) $2x^2 - 3px + 36 = 0$

Roots are $\alpha, 2\alpha$

(i) Product of roots

$= 2\alpha^2 = \frac{36}{2}$ ✓

$\alpha^2 = 9$

$\alpha = 3 \quad (\alpha > 0)$

(ii) subst $\alpha = 3$ into the equation

$18 - 9p + 36 = 0$

$54 = 9p$

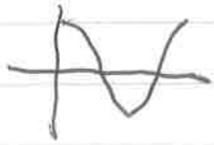
$p = 6$ ✓

[OR use Σ roots]

(f) $\cos\theta - 2\cos\theta\sin\theta = 0$

$\cos\theta(1 - 2\sin\theta) = 0$ ✓

$\cos\theta = 0$ or $\sin\theta = \frac{1}{2}$



$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

✓ ✓

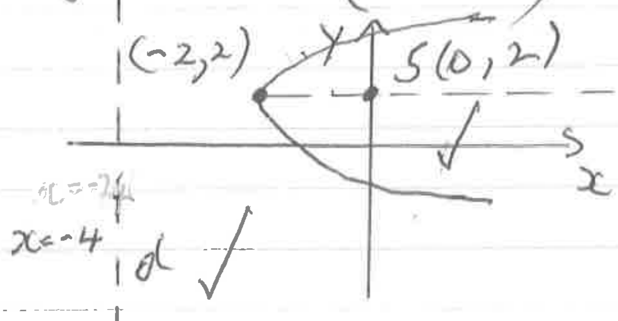
(e) $y^2 - 4y = 8x + 4$

$y^2 - 4y + 4 = 8x + 8$

$(y-2)^2 = 4(2)(x+2)$ ✓

focal length = 2

Vertex = (−2, 2) ✓



✓ ✓

5 (b)

FIFTEEN

15

(a) $A = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

$A \int_1^3 2^{x^2}\,dx \doteq \frac{3-1}{6}\left[f(1) + 4f(2) + f(3)\right]$

$f(1) = 2$, $f(2) = 16$, $f(3) = 512$ ✓

$\doteq \frac{1}{3}(2 + 4 \times 16 + 512)$

$\doteq \frac{578}{3} \doteq 192.6\ u^2$ ✓

[Award 1 wrong values entered into correct formula]

(b) $x = \frac{t^3}{3} - \frac{t^2}{2} - 6t + 11$

$\frac{dx}{dt} = t^2 - t - 6$

Velocity zero when

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t = 3$ or $t = -2$ ✓

when $t = 0$, $x = 11$

$t = 3$, $x = 9 - \frac{9}{2} - 18 + 11$

$= 20 - 22\frac{1}{2}$

$= -2\frac{1}{2}$ ✓

$t = 4$, $x = \frac{64}{3} - 8 - 24 + 11$

$= \frac{1}{3}$

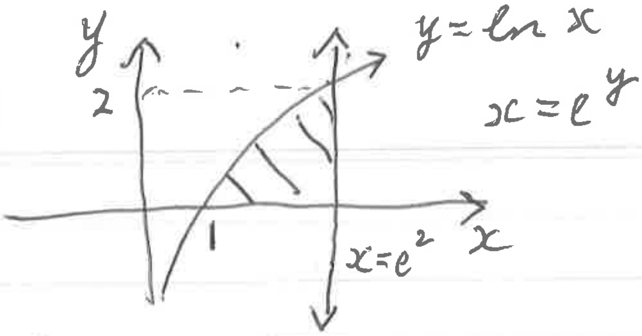
In first 4 secs average speed $= \frac{\text{distance travelled}}{4}$

$= \frac{13\frac{1}{2} + 2\frac{1}{2} + \frac{1}{3}}{4}$ ✓

$= 4\frac{1}{12}$ m/sec

($\doteq 4.083$) Accept approx

(c) (i)



✓ graph of $y = \ln x$
✓ correct region

(ii) $V = \pi \int_0^2 x_R^2 - x_L^2 \, dy$ ✓

$= \pi \int_0^2 e^4 - e^{2y} \, dy$

$= \pi \left[e^4 y - \frac{e^{2y}}{2} \right]_0^2$ ✓

$= \pi \left(\left(2e^4 - \frac{e^4}{2} \right) - \left(0 - \frac{1}{2} \right) \right)$

$= \pi \left(\frac{3e^4}{4} + \frac{1}{2} \right)$ units^3 ✓

(d) (i) $A_1 = 1.0025M$ ✓

$A_2 = 1.0025M + 1.0025^2 M$

(ii)

$A_3 = 1.0025M + 1.0025^2 M + 1.0025^3 M$

$A_n = (1.0025 + 1.0025^2 + \ldots 1.0025^n M$ ✓

$= M \times \frac{a(r^n - 1)}{r - 1}$

$= \frac{M \times 1.0025(1.0025^n - 1)}{0.0025}$

✓ $= 401M(1.0025^n - 1)$ ✓

$A_{24} = 401M(1.0025^{24} - 1) = 15000$

$M = \$606$ (to nearest dollar

✓

SIXTEEN

T5

(a)

(i) $\frac{dP}{dt} = kP$

$P = P_0 e^{kt}$

$P_0 = 12500$

$t = 15 \quad P = 16000$ ✓

$16000 = 12500 e^{k(15)}$

$\therefore k = \frac{1}{15} \ln \frac{32}{25}$ ✓

(ii) When $t = 20$

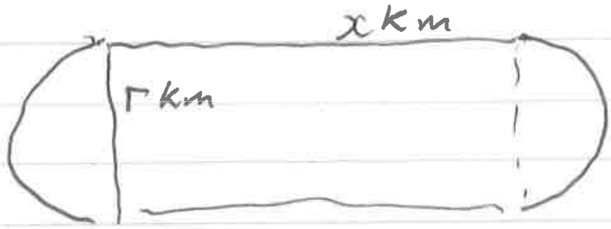
$P = 12{,}500\, e^{\frac{4}{3} \log_e\left(\frac{32}{25}\right)}$ ✓

$= 12{,}500 \times \left(\frac{32}{25}\right)^{4/3}$

$\doteq 17370$ (nearest 10) ✓

(Accept correct to the nearest hundred)

(b)



(i) Perimeter $= 2\pi r + 2x = 1$

$x = \frac{1 - 2\pi r}{2}$ ✓

(ii) $A = 2r\,x$

(Rectangle) $= 2r\left(\frac{1 - 2\pi r}{2}\right)$ ✓

$= r - 2\pi r^2$ (upside parabola so has max)

(iii) $A' = 1 - 4\pi r$ ✓

$A'' = -4\pi < 0$

Max where $A' = 0$

$1 - 4\pi r = 0$ ✓

$r = \frac{1}{4\pi}$ $\quad x = \frac{1}{4}$ ✓

7b

T5

SIXTEEN

(a)

(i) $\frac{dP}{dt} = kP$

$P = P_0 e^{kt}$

$P_0 = 12500$

$t = 15 \quad P = 16000$ ✓

$16000 = 12500\, e^{k(15)}$

$\therefore k = \frac{1}{15} \ln \frac{32}{25}$ ✓

(ii) When $t = 21$

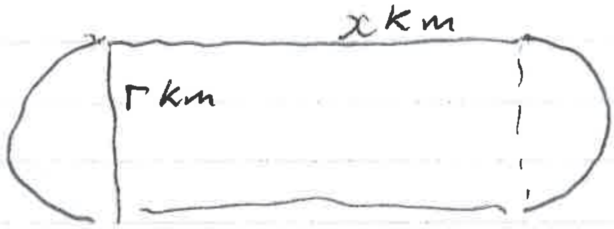
$P = 12{,}500\, e^{\frac{21}{15} \log_e\left(\frac{32}{25}\right)}$ ✓

$= 12{,}500 \times \left(\frac{32}{25}\right)^{\frac{21}{15}}$

$\doteq 17660$ (nearest 10) ✓

(Accept correct to the nearest hundred)

(b)



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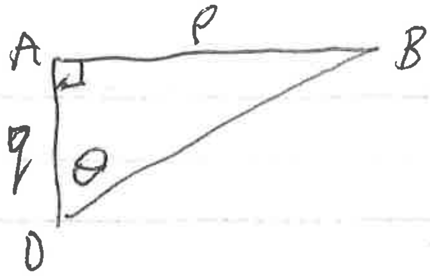
$r = \frac{1}{4\pi}$ $\quad x = \frac{1}{4}$ ✓

(c) (i)



By Pythagoras'

$BD = \sqrt{p^2+q^2}$ ✓

$\cos\theta = \dfrac{q}{\sqrt{p^2+q^2}}$

(ii) Area Δ AD $= \frac{1}{2} p q$ ✓

Also Area $\Delta ADB = \frac{1}{2} AE \times DB = \frac{1}{2} pq$

$AE = \dfrac{pq}{DB} = \dfrac{pq}{\sqrt{p^2+q^2}}$ ✓

(There are other ways to do this)

(iii) $AG = \frac{1}{2} AC = \frac{1}{2}\sqrt{p^2+q^2}$ (Pythagoras)

$EG^2 = AG^2 - AE^2$

$= \frac{1}{4}(p^2+q^2) - \dfrac{p^2q^2}{p^2+q^2}$ ✓

$= \dfrac{p^4+2p^2q^2+q^4-4p^2q^2}{4(p^2+q^2)}$

$= \dfrac{p^4-2p^2q^2+q^4}{4(p^2+q^2)}$

$EG^2 = \dfrac{(p^2-q^2)^2}{4(p^2+q^2)}$ ✓

$EG = \dfrac{|p^2-q^2|}{2\sqrt{p^2+q^2}} = \dfrac{p^2-q^2}{2\sqrt{p^2+q^2}}$ since $p > q$

Area $\Delta AEG = \frac{1}{2} AE \times EG$

$= \frac{1}{2} \dfrac{pq}{\sqrt{p^2+q^2}} \times \dfrac{(p^2-q^2)}{2\sqrt{p^2+q^2}}$ ✓

$= \dfrac{pq(p^2-q^2)}{4(p^2+q^2)}$